The effect of contour concentricity on the acceleration sensitivity of quartz crystal resonators

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Abstract— **Optimization of the acceleration sensitivity of quartz crystal resonators has been a challenging problem for resonator designers for decades. The structural symmetry of the resonator and mount combination has been shown in past work, both theoretical and practical, to have a strong influence on acceleration sensitivity, and specialized structures have been developed [1], [2], [3] that have greatly improved performance. However, with applications such as airborne radar systems, there is a persistent demand for further improvement.**

The design of many of the practical high-stability resonator products that have a need for good acceleration sensitivity is also constrained by other attributes, such as high quality factor, and these constraints typically result in a low-frequency overtone device with a fully contoured resonator element design. In this paper, the effect of the concentricity of the contour shape on the quartz disk in contoured resonators is considered, and results are presented that demonstrate a strong correlation between the contour offset from the blank center and the acceleration sensitivity of the resonator. Methods are also described for measurement of the contour position relative to the perimeter of the disk.

Keywords—Quartz crystal, resonator, contoured, sensitivity, acceleration, g-sensitivity, optical measurement.

I. BACKGROUND

The acceleration sensitivity, sometimes called g-sensitivity, of crystal resonators has been widely discussed over the past few decades. The parameter is most important in applications that require good phase noise, but where the device is exposed to high vibration fields. A good example would be a frequency reference for a radar system in a helicopter. Theoretical work by Tiersten and Zhou and others essentially concluded that a quartz resonator with perfect spatial symmetry in both the resonator element and the mounting structure will exhibit zero g-sensitivity [5],[6],[7],[8].

Theoretical and practical work by Eernisse and colleagues [9],[10],[11],[12] proposed and implemented practical mount designs to approximate symmetric structures with the aim of achieving low acceleration-induced frequency shifts. They also looked at the use of carefully positioned masses deposited onto the surface of blanks to modify the position of the resonance mode and hence to improve g-sensitivity. This technique may be very useful for planar, higher frequency resonator designs, but in typical lower frequency, low phase-noise designs, the blank geometry is by necessity contoured, and in these cases incremental mass loading changes on the electrode surface have very little influence on the mode shape. Many other authors have discussed practical and theoretical considerations for achieving low acceleration sensitivity, including Kosinsky [13],[14] and Lee [15]. Practical designs were also developed in France from the 1970s onwards in the form of the highly complex BVA structures that utilize quartz bridges in the resonator elements as well as multiple quartz components and conductive structures to provide symmetrical support for the resonator [16],[3].

Haskell *et al* [1],[2],[4] introduced the patented Quad Relief Mount product or QRM. This uses a planar mount design configuration that is positioned to coincide with the central plane of the resonator element as shown in Fig 1. The outer ring is a rigid ceramic structure that is firmly attached to the crystal base and in between the ring and the blank is an essentially planar array structure that also provides a reduction in static stresses to the resonator. This design approach has achieved excellent performance, with g-sensitivity results below 10^{-10}/g in some cases, while still performing well for Q and phase noise. However, as is often the case with crystal parameters, there is typically a distribution in performance for g-sensitivity in each manufactured group, and this causes yield problems as well as unpredictability in production scheduling. The work reported here aimed to find the root cause of these anomalous results, with a focus on the contour concentricity.

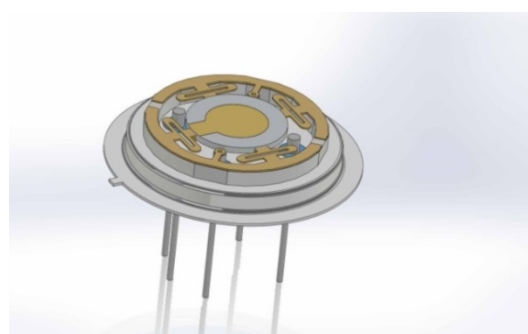

Fig 1 QRM mount structure

II. EXPERIMENTAL APPROACH

The focus for this work was a typical QRM resonator type that is currently manufactured: a 10MHz 3rd overtone SC cut for ovenized oscillator application. The design uses a planoconvex blank geometry with a contour of approximately 1.5 diopters on the convex side. Rather than deliberately manufacturing units with known asymmetries and then measuring them for acceleration sensitivity, the approach that was used was to select parts from past groups with a range of performance. The units were re-measured to verify g-sensitivity results and then inspected for manufacturing anomalies. Finally the blanks were removed from the mounts and the electrodes stripped to allow analysis of the contour.

III. Contour offset measurement methods

The geometry of a spherically contoured surface is shown in Fig 2. Historically, because the machining processes in quartz crystal manufacture were derived from methods used in the optical lens industry, the radius of curvature is often specified in diopters. Strictly, this parameter is only defined for a medium with a known refractive index and, as described later, the refractive index is not well defined for crystalline quartz, so the index for crown glass of 1.525 is usually substituted, which results in the relationship $R = 525/D$ where R is measured in mm and D is the diopter value.

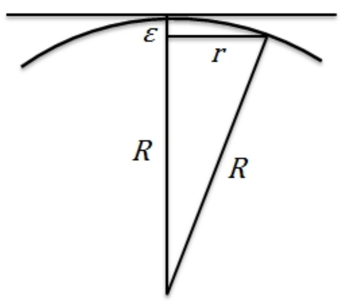

Fig 2 Contour geometry

To derive the relationship between the radius of curvature R and incremental thickness change ε at an offset radius r, application of Pythagoras gives:

$$(R - \varepsilon)^2 + r^2 = R^2$$

And omitting the ε^2 term gives

$$\varepsilon \approx r^2/_{2R}$$

There are various viable methods for measurement of the concentricity between the blank periphery and the contour surface which determines the mode position of the resonance. There are pros and cons for each technique depending on the geometry being observed, so the methods were evaluated for the particular blank geometry used in this product.

A. Pre-electroding

One potential option for measurement of the contour offset of a blank is based on the relationship between the electrode location relative to the blank geometry and the resulting motional parameters of the resonator. To determine the relationship between contour center position and the resulting motional capacitance C1 of a resonator, a model was created using the structural mechanics module of Comsol Multiphysics. An example of a 3D plot showing displacement intensity for the 10 MHz 3rd overtone SC cut resonator used in this study is shown in Fig 3. The model was set up with varying contour offset, and the resulting relationship between C1 and contour offset is shown in Fig 4 for various electrode diameters. Clearly this relationship could also be derived analytically, but Comsol has proven to be a very useful tool for this type of calculation.

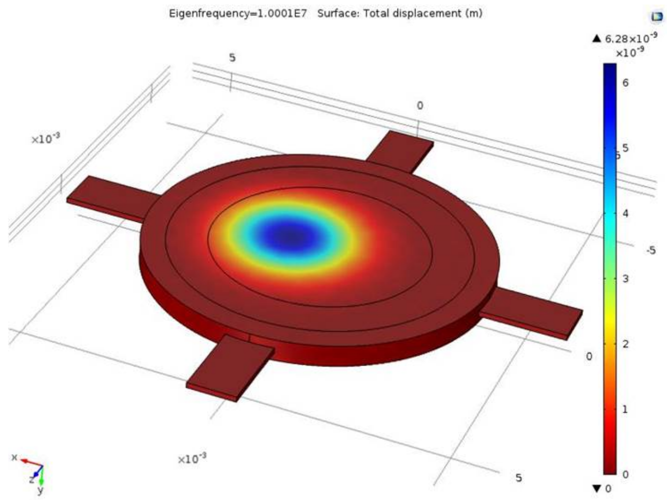

Fig 3 Contour map of resonator with offset contour

To sort a group of contoured blanks using this method, they would first be checked for contour radius, since this is clearly also a parameter that strongly influences C1. They would then be accurately plated with small circular electrodes in the center of the blanks, preferably with an electrode material that is easily removed, and then inserted into temporary mounts. The optimum size of the electrode depends on the design being analyzed. A simple motional parameter check would then provide the tool to select for good contour concentricity, after which the electrodes would be removed and then the parts re-processed into the final product.

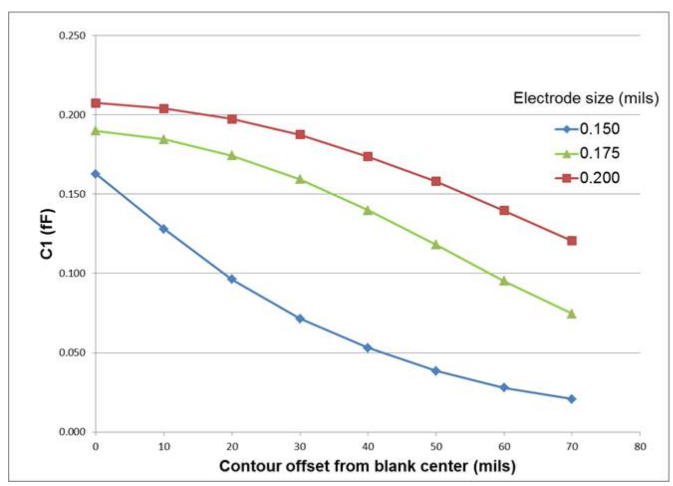

Fig 4 C1 vs. contour offset for various electrode diameters

B. Profile measurement

Another method that can be considered to measure contour concentricity is a 1D or 2D profile measurement, preferably using a non-contact approach. Various methods have been used for non-contact contour measurement of blanks, such as a laser triangulation or a confocal type of depth gauge. A typical output plot from such a system is shown in Fig 5, which in this case includes a least-squares fit to a circular arc. In the standard process, this fitted curve is used to calculate contour radius. This plot illustrates that the blank is also beveled on the contoured side.

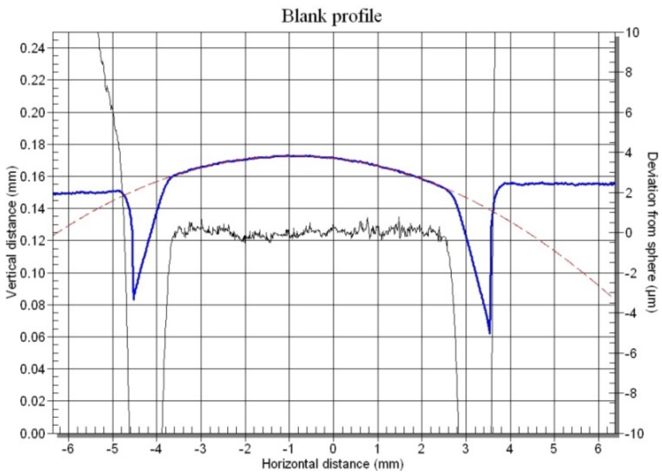

Fig 5 Profilometer plot with fitted curve

In the measurement shown here, the blank is inserted into a fixture with a flat upper surface and an accurately machined pocket in which the quartz disk is placed. The flat surface provides a reference line in the plot, as well as reference edges that indicate the perimeter of the blank. After mathematically compensating for the slope of the reference surface, the center of the fitted curve relative to the pocket edges should represent the contour offset.

In practice, this method cannot differentiate between contour offset and physical tilt of the blank (for example due to a particle under one side of the disk), identifying the blank edge is difficult, and to characterize a blank fully, multiple scans are required. So although it is a potentially useful method, the inherent sources of inaccuracy need to be considered.

C. *Use of optical properties of quartz - birefringence*

A property of quartz that can be useful for various measurement techniques throughout crystal manufacture is its anisotropic optical characteristic of *birefringence*, a property that is exhibited to a varying degree by all transparent media with non-cubic crystalline structures. Birefringence is characterized by a refractive index that depends on the propagation direction or polarization direction of light passing through it. The simplest form of birefringence is described as *uniaxial*, which means that rotation about one axis does not affect the passage of light passing through the medium. This single axis is called the *optic axis*, and light for which the polarization direction is perpendicular to the optic axis is called an *ordinary ray*, and it exhibits a refractive index of n_O. Light with a polarization direction parallel to the optic axis is called an *extraordinary ray*, and its refractive index is denoted n_E. Quartz has three axes of two-fold symmetry and one axis of three-fold symmetry; this form of crystal structure is classed as having trigonal symmetry, and materials such as quartz with this form exhibit uniaxial birefringence. The axis of three-fold symmetry is conventionally denoted the Z-axis, and this is the optic axis for quartz. The two discrete refractive index values at various wavelengths through and beyond the visible range are shown in table 1 [17].

Table 1 Refractive index of quartz at different wavelengths

λ (nm)	n_O	n_E
231	1.6140	1.6256
340	1.5675	1.5774
394	1.5585	1.5681
434	1.5540	1.5634
508	1.5482	1.5575
589	1.5442	1.5534
768	1.5390	1.5479
833	1.5377	1.5466
991	1.5351	1.5439
1159	1.5328	1.5415

The measurement method uses full-spectrum white light in transmission through the sample, and two linear polarizing filters are placed above and below the blank being measured. The polarizing filters are set up with the polarization directions at right angles to each other so that the background is dark. The plate orientations of any of the rotated cuts that are typically used as resonators have components along both the optic axis and perpendicular to it, so light passing through the quartz will experience two distinct velocities, as defined by the two refractive indices. The resulting effect is a rotation in the polarization of the light that is a function of the blank thickness and the wavelength of the light, and this causes a range of colors to be observed in the transmitted light.

To quantify the effect, consider the graphic in Fig 6.

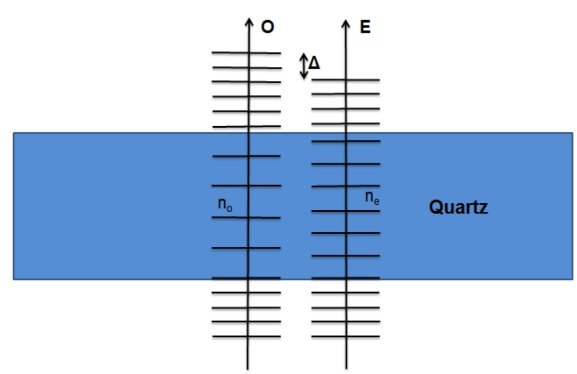

Fig 6 Illustration of the effect of birefringence through quartz

The separation distance Δ between wavefronts of the two rays is given by:

$$\Delta = c(t_E - t_O)$$

where c is the speed of light in free space and t_E and t_O are the propagation times for the two rays, so

$$\Delta = c\left(\frac{h}{v_E} - \frac{h}{v_O}\right) = h\left(\frac{c}{v_E} - \frac{c}{v_O}\right)$$

where h is the thickness of the medium and v_E and v_O are the velocities of the rays.

But c/v is the refractive index n of the material, so

$$\Delta = h(n_E - n_O)$$

The absolute difference $n_E - n_O$ between the two refractive indices is the definition of the birefringence of the material, and for quartz at a typical visible wavelength,

$$\Delta \approx 0.009\, h$$

The resulting image exhibits bright lines where the value of Δ corresponds to integral wavelength multiples for the wavelength of the light being passed, but because of the range of wavelengths in the visible spectrum, a color pattern is observed, as shown in the Michel-Levy birefringence chart in Fig 7.

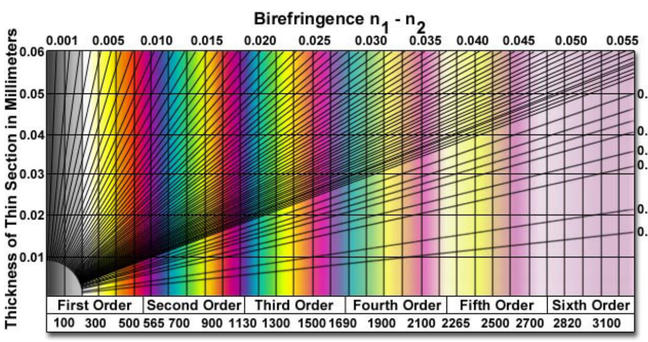

Fig 7 Michel-Levy birefringence chart

For the 10MHz blank in question, the resulting first ring radius would be about 1.3mm, and this may work well in a metrology system, but in practice when using this method the contrast is fairly poor in the center region where the required accuracy is highest. It works very well for blanks with steeper contours, and can be used on blanks without polished surfaces. A 10MHz fundamental blank with a contour of 10D would exhibit a ring with a radius of 0.4mm.

D. Newton's rings using monochromatic light

Another simple method to view incremental thickness variations of a plate of a transparent medium is to expose the plate to monochromatic light from one side, normal to the plate. If both surfaces are specular, the light is reflected from the top and bottom surfaces, resulting in interference patterns known as Newton's rings. The method is often used on convex lenses in conjunction with an optical flat, where the lens is placed such that there is an air gap between the flat and the part being observed, but, as in this case, it also works for thin transparent lenses where the reference surface is the other face of the lens.

In this measurement method, the thickness increments between light and dark bands are directly related to the wavelength of the light being used. In our configuration the light source was a low-pressure sodium lamp that has a spectral line pair at 590nm and 590.6nm. The thickness increment ε is given by:

$$2\varepsilon = \frac{\lambda}{n}\left(m - \frac{1}{2}\right)$$

where λ is the light wavelength in free space, n is the nominal refractive index of the medium and m is the ring number. The radius of the observed rings r_m is then given by:

$$r_m = \sqrt{\frac{R\lambda}{n}\left(m - \frac{1}{2}\right)}$$

In the crystal design investigated in this study, the first ring would occur at a radius of about 0.25mm, which gives very good resolution of the contour location. The set up to measure blanks using this method employs a sodium lamp from which the light is directed through a beam splitter to the part being tested, and then the image is viewed through a low-power microscope using a Scienscope Smartcam; this system provides a very simple method for measurement of the distance between two circles. The main disadvantage of this technique is that it requires highly polished surfaces.

IV. RESULTS

A group of 50 finished, encapsulated QRM resonators was measured in three axes for g-sensitivity using the passive method described in 2003 [18]. They were selected from past groups based on good performance in other respects such as Q and C1, to avoid otherwise anomalous parts being included in the population. The packages were opened, the blanks removed and the gold electrodes were stripped with aqua regia, taking great care to retain the individual identity of each blank.

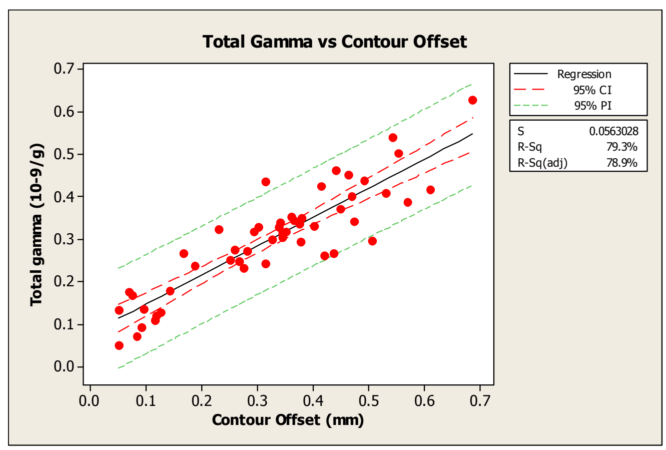

Fig 8 g-sensitivity versus contour offset for measured group

The group was then measured for contour concentricity in magnitude and direction, and the scatter plot between contour offset and the magnitude of the g-sensitivity vector is shown in Fig 8. Although both g-sensitivity and contour offset were measured as vector quantities, the best correlation was obtained between the magnitudes of the two parameters as shown. Some scatter can be expected in this plot because the magnitudes and not the directions are taken into account for either parameter. Also other mounting irregularities were not taken into consideration, but nevertheless, the overall trend is very conclusive.

V. Conclusion

Several methods have been identified that can be used to measure the concentricity of the contour shape in a quartz resonator blank. An experiment was set up on a 10MHz 3rd overtone QRM product to compare the contour offset with the measured values of acceleration sensitivity. As expected for this product type, there is a clear correlation between the two parameters. The positive result has prompted a series of developments to improve contour machining processes, and has resulted in significant improvements in product quality and yield.

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