# METHOD FOR MEASUREMENT OF THE SENSITIVITY OF CRYSTAL RESONATORS TO REPETITIVE STIMULI

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***Abstract* - This paper describes a simple method for the measurement of the effect of periodic stimuli on the frequency of crystal resonators. The technique has been implemented for the measurement of both acceleration and pressure sensitivity. The system is based around a passive method that has been previously used for residual phase noise measurement. In the case of acceleration sensitivity measurement, the crystal is placed on a vibration table. In this case a sinusoidal acceleration is applied to the crystal in each of three mutually orthogonal axes. To measure pressure sensitivity, a simple chamber has been built in which the barometric pressure is modulated around standard atmospheric conditions by about ±80mbar. For typical high Q 10 MHz 3rd overtone quartz resonators, the measurement resolution that has been achieved by the system is in the low parts in $10^{12}$ per g or about $10^{-9}$ per bar. The measurement method is relatively fast, and it obviates the need to stabilize precisely the temperature of the resonator.**

## I. INTRODUCTION

In recent years significant progress has been made in crystal resonator design to effect a substantial reduction in g-sensitivity values [1]. With such reduced levels of acceleration sensitivity, the need has arisen for a measurement system with appropriately higher resolution.

The most obvious method for determination of g-sensitivity for a stable ovenized oscillator is the '2g tipover test', where the whole oscillator is simply inverted, resulting in an incremental 2g change in the internal forces applied to the resonator [2]. However, since the changes are in the order of $1.10^{-9}$ per g, the oscillator temperature must be pre-stabilized, and this can be a long process. With this method, care must also be taken to avoid convectional temperature effects in the oven cavity, which can cause misleading results.

In this work, for g-sensitivity measurement, the method that has been implemented is based on the imposition of an essentially sinusoidal low frequency vibration field on the resonator by placing the device on a vibration table, and then observing the modulation effects on the resonator frequency.

The method is applicable to any repetitive stimulus to which the frequency of the resonator exhibits sensitivity. Such stimuli could include pressure, electromagnetic radiation effects, magnetic fields, neutron radiation, and so on.

Pressure sensitivity has also been a focus of recent work, since for crystals that are to be used in the highest precision oscillators, the effect of changes in barometric pressure can be of the same order of magnitude as the tolerances for aging and temperature variation. This effect can impact the performance of the product as well as causing confusion in the oscillator manufacturing process and characterization.

## II. THEORY OF G-SENSITIVITY MEASUREMENT

It is well known that, to a good approximation, if an acceleration $a$ is applied to a crystal resonator, then the result is a frequency change which is proportional to this acceleration. The acceleration sensitivity Γ is dependent on the direction of the acceleration, and can be considered a vector quantity. The measurements are typically performed with equal accelerations in three mutually orthogonal axes, so the components of Γ are measured as scalar quantities, and the spatial properties of Γ can then be derived from the three components.

When a low frequency sinusoidal vibration is applied to the crystal, the crystal frequency is modulated as a function of the vibration frequency [2]. In this case low frequency effects caused by thermal drift become insignificant, so the measurement can be performed relatively quickly.

The simplest condition for determination of the relationship between the modulation level and the g-sensitivity is with the crystal integrated into an oscillator circuit. Here, the effect is simply frequency modulation.

The instantaneous frequency can be described by:

$$w(t) = w_0 \{1 + g\, a \cos(w_v t)\}$$

where $w_v$ is the vibration frequency
$w_0$ is the resonator frequency
and $g$ is the component of the g-sensitivity vector in the direction of interest.

The oscillator output voltage can be represented by

$$v(t) = v_0 \cos\{f(t)\},$$

where the phase value is the time integral of frequency:

$$f(t) = w_0 t + g\, a \frac{w_0}{w_v} \sin(w_v t)$$

so

$$v(t) = v_0 \cos\left\{ w_0 t + g\, a \frac{w_0}{w_v} \sin(w_v t) \right\}$$

This is a cosine sine function, which can be expanded by using Bessel functions to determine the relative sideband levels for each of the vibration-induced sidebands.

Assuming the modulation index to be very small for this effect, this results in the well-known expression:

$$g = \frac{2w_v}{aw_0} 10^{L/20}$$

where $L$ is the relative sideband level in dBc for the first order sideband.

For crystal measurement, an alternative approach can be used, where the crystal is placed in a passive network, and driven by an unmodulated frequency at the series resonance [3, 4]. The modulation induced in this case is essentially phase modulation.

The phase slope is related to the loaded Q, $Q_L$ , by the expression:

$$Q_L = \frac{w_0}{2} \frac{df}{dw}$$

Since $$\frac{dw}{w_0} = g\,a \cos(w_v t)$$

it follows that:

$$v(t) = v_0 \cos\{w_0 t + 2Q_L a g \cos(w_v t)\}$$

and for a small modulation index,

$$g = \frac{10^{L/20}}{aQ_L}$$

However, the resulting sidebands are attenuated by the amplitude response of the crystal network, which is a bandpass filter with the transfer function:

$$G(w) = \frac{1}{\sqrt{1 + \left(\frac{2Q_L w_v}{w_0}\right)^2}}$$

The g-sensitivity is now given by:

$$g = \frac{2w_v}{w_0 a} \sqrt{1 + \left(\frac{w_0}{2Q_L w_v}\right)^2}\, 10^{L/20}$$

or

$$g = \frac{\sqrt{4w_v^2 + w_3^2}}{w_0 a} 10^{L/20}$$

where $w_3$ is the 3dB bandwidth of the crystal network. Note that for high Q resonators, in which case $w_3 << w_v$, the expression becomes identical to the oscillator case. This is confirmed by practical measurements, where the sideband levels are essentially the same for a resonator when measured by the passive and oscillator methods.

## III. MEASUREMENT USING NETWORK / SPECTRUM ANALYZER

A number of instruments are available that can be configured as spectrum analyzers with very low resolution-bandwidths of about 1Hz. In this work we have used the HP4396B, which also can be configured as a network analyzer.

The measurement system is set up as shown in Fig. 1. The instruments are controlled via the IEEE 488 interface, and a Visual Basic program has been written to perform the measurement. Both analyzer and synthesizer are connected to the house rubidium frequency standard, so that there is no significant frequency error between the two instruments. In this configuration, either the output of the analyzer can be enabled or that of the synthesizer. This allows the crystal network transmission response to be analyzed as a first step (with the swept analyzer output enabled). This is used to determine the loaded Q and the series resonant frequency. The analyzer output is then disabled and the fixed synthesizer output is set to the crystal frequency.

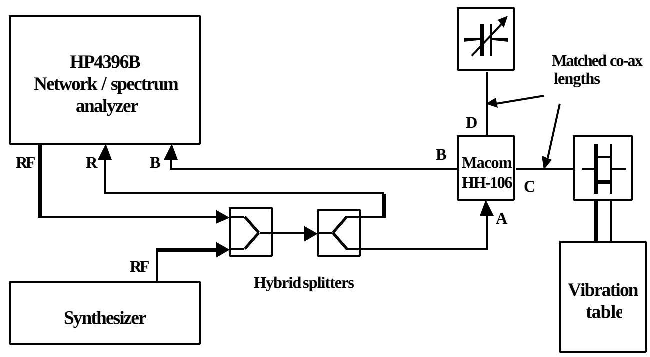


**Fig. 1.** Schematic of passive system for g-sensitivity measurement using synthesizer and spectrum analyzer

With the crystal under vibration at a frequency $f_v$, the resulting spectrum consists of the synthesizer output frequency $f_0$ together with upper and lower sidebands at $f_0 + f_v$ and $f_0 - f_v$, in addition to a series of higher order terms. Any imbalance between the upper and lower sidebands is typically caused by thermal drift of the crystal frequency, which results in the series resonance being offset from the synthesizer output frequency. The measured sideband imbalance, in addition to the change in attenuation of the carrier frequency, is used to adjust the synthesizer frequency automatically to track the crystal response.

This measurement approach has been used extensively for crystals with g-sensitivity in the range of a few parts in $10^{10}$ per g, but achievement of higher resolution requires a change in approach. The limitation is a function of the dynamic range of the analyzer, so one option to circumvent this problem is to use a frequency multiplier on the modulated signal [2]. Doubling the frequency of a signal with frequency or phase modulation increases the relative sideband levels by 6dB, so this gives a corresponding improvement in measurement resolution. This does, however, necessitate making a special purpose multiplier for each measurement frequency, and does not lend itself well to setting up a general-purpose measurement system.

Another limitation of this method is that the spectrum measurement only gives information on signal amplitude, and not on phase. Although the value of $\gamma$ in any one of the orthogonal measurement axes is a scalar quantity, it does have a sign, and this information is lost using this measurement method. The measurement is also rather slow, primarily because of the sweep time required by the analyzer to obtain a low resolution-bandwidth. Another problem that has been encountered with this method arises from the g-sensitivity of the analyzer itself. If there is any mechanical coupling between the vibration table and the instrument, this can cause misleading results.

## IV. MEASUREMENT USING PHASE DETECTOR AND LOCK-IN AMPLIFIER

An alternative measurement setup, which solves the problems of resolution, provides sign information, and also enables very fast measurements, is shown in Fig. 2. This equipment is also incorporated into the measurement setup under IEEE-488 control, and the Visual Basic program can optionally use this configuration or the network analyzer configuration or both.

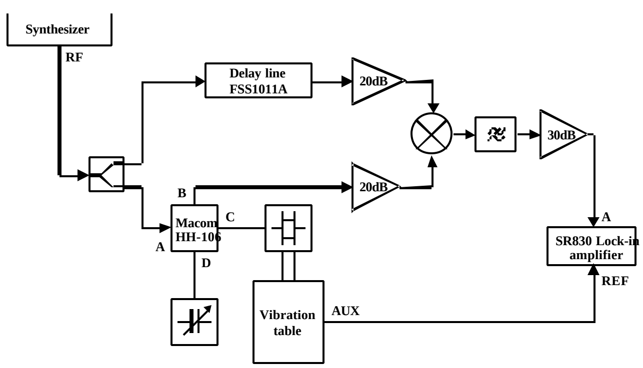


**Fig. 2.** Measurement setup with lock-in amplifier

The principle of this method is based around a commonly used residual phase-noise measurement technique [5, 6]. The source is a frequency synthesizer, set at the crystal resonant frequency. This signal is split with a Macom JH-10-4 quadrature splitter. The specified frequency range of this component is 20MHz to 140MHz, but in practice it can be used well outside this range at the cost of some attenuation. One of the signals is fed through the crystal network into one input of a phase detector. The other passes through a programmable delay line to the other phase detector input. The idea here is to provide a pair of matched line lengths so that the phase difference at the phase detector is 90° when the crystal is at series resonance. This could in principle be achieved with a simple hybrid splitter and a quarter-wavelength section of coaxial cable, but at 5 MHz this would require a physical cable length of over 8 meters. The quadrature splitter simplifies this task, and facilitates the general-purpose measurement approach. The line length in the reference path is adjusted as a function of frequency with the programmable delay line. This compensates for variations arising from the splitter, cables and crystal network, and is achieved by characterizing the system over the full frequency range, then implementing a sixth order polynomial curve fit to set the delay function. The phase detector and delay line that are used in this work are the Femtosecond Systems products FSS1000E and FSS1011A, which integrate well together.

The signal levels at the inputs of the phase detector should ideally be over +10dBm to set the phase detector into a condition with reasonable gain, so a pair of matched amplifiers is used for this purpose. In practice, the reference signal level is set at about +10dBm, which results in a lower level in the measurement signal because of the attenuation of the crystal network, but this configuration still gives adequate phase detector gain.

The resulting baseband voltage is measured with a lock-in amplifier, and the reference frequency for the measurement is derived from the auxiliary output of the vibration table. This instrument acts as a very narrow bandwidth synchronous filter and is very good at extracting small signals of known frequency from noise [7, 8, 9].

In this setup, the acceleration sensitivity for high Q resonators is determined by:

$$g = \frac{V_{rms}\sqrt{2}w_v}{K_d K_a a w_0}$$

where $V_{rms}$ is the measured baseband voltage
$K_d$ is the phase detector gain
$K_a$ is the amplifier gain (in this case 30dB)
$a$ is the peak acceleration in g
$w_v$ is the vibration frequency
and $w_0$ is the crystal frequency

For lower Q resonators, the equation becomes:

$$g = \frac{V_{rms}\sqrt{2}w_v}{K_d K_a a w_0}\left(1 + \frac{w_3^2}{4w_v^2}\right)$$

where $w_3$ is the 3dB bandwidth of the crystal network.

The measurement program first adjusts the synthesizer frequency to series resonance, by an iteration process using the phase detector output voltage as a control parameter.

The phase detector gain $K_d$ is strongly dependent on the signal level through the crystal network, which in turn is dependent on the crystal resistance, and therefore is variable from measurement to measurement. Several options have been investigated to solve this problem, and the final solution is to measure the phase detector gain for each crystal. This is easily achieved by varying the line length in the delay line box to swing the phase about ±18° (±π/10) either side of quadrature, then measuring the change in the output voltage of the phase detector. This variation is still within the linear range of the phase detector, with enough resolution to determine the gain with sufficient accuracy.

The resonator Q is then measured by changing the synthesizer frequency either side of resonance (again staying within the linear phase detector range), the phase slope being determined by measurement of the change in the phase detector output voltage.

Finally, the synthesizer frequency is locked on resonance, and the g-sensitivity is measured by observation of the lock-in amplifier voltage. Checks have been added to ensure that the lock-in is not overloaded, and that the reference frequency is locked to the vibration frequency. The phase of the signal at the lock-in input is used to determine the sign of the acceleration sensitivity. The whole measurement sequence takes about 20 seconds.

## V. EXTENSION TO PRESSURE-SENSITIVITY MEASUREMENT

The measurement of pressure sensitivity is somewhat analogous to vibration sensitivity measurement. The analog to the 2-g tipover test could comprise a chamber, which encloses an oscillator into which the crystal is incorporated, and which can be modulated in pressure to a number of discrete values, usually two calibrated pressures. The variation in oscillator frequency is then observed to determine the effect of pressure. As for the 2-g tipover test, this method necessitates the use of an ovenized oscillator to avoid the effects of longer-term drift, which superimposes on the observed variation due to pressure. Again, this stabilization process can take a long time.

To derive a test approach that uses a repetitive stimulus for pressure sensitivity measurement, a test system has been built that comprises a modified diaphragm pump and a simple hermetic chamber. The pump is a dual chamber type with a chamber diameter of about 55mm. The stroke is set by an eccentric cam, of which several variants have been made. The final choice gives a 2.8mm stroke, and the locations of the cams are set so that the diaphragms move in phase. In the standard pump, a pair of reed valves enables the air to be pushed through the cavity to result in pumping action. In this setup, one of the valves in each chamber is blocked completely with epoxy, and the other is removed to leave the port continuously open, with this port connected to the measurement chamber. Thus a fixed amount of air is contained in the whole system, and the pump diaphragm

TABLE I
TYPICAL G-SENSITIVITY RESULTS SHOWING BOTH NETWORK ANALYZER AND LOCK-IN AMPLIFIER MEASUREMENT METHODS

| Device # | ($10^{-9}$ /g) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $\Gamma_x$ network | $\Gamma_x$ lock-in | $\Gamma_y$ network | $\Gamma_y$ lock-in | $\Gamma_z$ network | $\Gamma_z$ lock-in | $\lvert\Gamma\rvert$ network | $\lvert\Gamma\rvert$ lock-in |
| Mini-8NF-DD-QRM-01 | 0.168 | -0.177 | 0.065 | 0.054 | 0.282 | 0.272 | 0.335 | 0.329 |
| Mini-8NF-DD-QRM-02 | 0.027 | -0.020 | 0.337 | 0.338 | 0.138 | -0.116 | 0.365 | 0.358 |
| Mini-8NF-DD-QRM-03 | 0.042 | 0.036 | 0.141 | -0.138 | 0.430 | -0.393 | 0.454 | 0.418 |
| Mini-8NF-DD-QRM-04 | 0.088 | -0.077 | 0.018 | -0.015 | 0.025 | -0.037 | 0.093 | 0.087 |
| Mini-8NF-DD-QRM-05 | 0.117 | -0.122 | 0.340 | 0.353 | 0.680 | -0.630 | 0.769 | 0.732 |
| Mini-8NF-DD-QRM-06 | 0.153 | -0.170 | 0.210 | -0.229 | 0.568 | 0.548 | 0.625 | 0.618 |
| Mini-8NF-DD-QRM-07 | 0.141 | -0.138 | 0.089 | -0.078 | 0.125 | -0.122 | 0.208 | 0.200 |
| Mini-8NF-DD-QRM-08 | 0.044 | -0.043 | 0.064 | -0.055 | 0.028 | -0.028 | 0.083 | 0.075 |
| Mini-8NF-DD-QRM-09 | 0.186 | 0.203 | 0.150 | -0.155 | 0.264 | 0.246 | 0.356 | 0.355 |
| Mini-8NF-DD-QRM-10 | 0.180 | 0.177 | 0.272 | -0.227 | 0.400 | -0.308 | 0.516 | 0.422 |
| Mini-8NF-DD-QRM-11 | 0.200 | -0.201 | 0.115 | -0.114 | 0.070 | 0.072 | 0.241 | 0.242 |
| Mini-8NF-DD-QRM-12 | 0.110 | -0.098 | 0.064 | -0.052 | 0.158 | -0.124 | 0.203 | 0.166 |
| Mini-8NF-DD-QRM-13 | 0.442 | -0.457 | 0.059 | 0.061 | 0.085 | -0.076 | 0.454 | 0.467 |
| Mini-8NF-DD-QRM-14 | 0.156 | -0.150 | 0.037 | -0.028 | 0.248 | 0.237 | 0.295 | 0.282 |
| Mini-8NF-DD-QRM-15 | 0.218 | 0.229 | 0.023 | 0.023 | 0.288 | 0.278 | 0.362 | 0.361 |
| **Average** | | | | | | | **0.357** | **0.341** |
| | | | | | | | | |
| 10NF-DD-QRM-01 | 0.410 | 0.309 | 0.270 | 0.119 | 0.290 | -0.394 | 0.570 | 0.515 |
| 10NF-DD-QRM-02 | 0.080 | -0.051 | 0.260 | -0.341 | 0.600 | -0.678 | 0.659 | 0.761 |
| 10NF-DD-QRM-03 | 0.100 | -0.064 | 0.260 | 0.142 | 0.070 | -0.139 | 0.287 | 0.209 |
| 10NF-DD-QRM-04 | 0.080 | 0.042 | 0.080 | -0.033 | 0.230 | -0.151 | 0.256 | 0.160 |
| 10NF-DD-QRM-05 | 0.150 | 0.023 | 0.110 | -0.047 | 0.080 | -0.114 | 0.202 | 0.125 |
| 10NF-DD-QRM-06 | 0.200 | 0.062 | 0.060 | -0.176 | 0.790 | 0.678 | 0.817 | 0.703 |
| 10NF-DD-QRM-07 | 0.340 | 0.154 | 0.260 | -0.200 | 0.360 | -0.540 | 0.559 | 0.596 |
| 10NF-DD-QRM-08 | 0.100 | -0.019 | 0.100 | -0.030 | 0.250 | 0.123 | 0.287 | 0.128 |
| 10NF-DD-QRM-09 | 0.190 | 0.064 | 0.090 | -0.067 | 0.320 | -0.399 | 0.383 | 0.410 |
| 10NF-DD-QRM-10 | 0.280 | 0.176 | 0.120 | -0.121 | 0.260 | -0.310 | 0.400 | 0.376 |
| 10NF-DD-QRM-11 | 0.170 | 0.025 | 0.130 | -0.017 | 0.160 | 0.031 | 0.267 | 0.043 |
| 10NF-DD-QRM-12 | 0.250 | 0.144 | 0.380 | 0.183 | 0.170 | -0.290 | 0.486 | 0.372 |
| 10NF-DD-QRM-13 | 0.120 | -0.015 | 0.100 | -0.162 | 0.180 | 0.168 | 0.238 | 0.234 |
| 10NF-DD-QRM-14 | 0.320 | 0.209 | 0.120 | -0.161 | 0.320 | 0.140 | 0.468 | 0.299 |
| 10NF-DD-QRM-15 | 0.160 | 0.015 | 0.170 | 0.025 | 0.110 | -0.142 | 0.258 | 0.145 |
| 10NF-DD-QRM-16 | 0.250 | 0.129 | 0.110 | -0.040 | 0.160 | 0.109 | 0.317 | 0.174 |
| **Average** | | | | | | | **0.403** | **0.328** |

simply modulates the pressure approximately sinusoidally. A pressure sensor is included in the system with a full-scale range of ±5psi (±0.320 bar), and the measured pressure variation is determined to be about ±80 mbar at a frequency of about 28Hz. The output of the pressure sensor is fed into the lock-in amplifier reference input instead of the output of the vibration table.

Some minor modifications have been made to the software, mainly to add a flag, which loads the appropriate defaults of pressure variation or acceleration level. The system has also been configured to track the stimulus frequency, which is useful for either measurement type.

## VI. TYPICAL RESULTS

The g-sensitivity measurement system described here has now demonstrated the ability to resolve down to parts in $10^{12}$ per g for typical 10 MHz 3rd overtone crystal types. Many devices have been measured for which the total $\Gamma$ is less than $2.10^{-10}$ per g, and some devices are in the mid-$10^{-11}$ range [1]. Individual components of this sensitivity have been measured in the low parts in $10^{12}$ per g. These measurements have been obtained using a 70 Hz stimulus of 10g peak acceleration.

Measurement of pressure sensitivity uses a stimulus frequency of around 30Hz, but only about 80 mbar peak pressure change. For a 10 MHz 3rd overtone resonator in the current setup, the limit in measurement resolution is about $1.10^{-9}$ per bar. This limit is partly caused by the phase noise of the synthesizer, which is currently an HP 3335, and is by no means optimized for phase noise at low offset frequencies. The resolution could be improved by an order of magnitude if a low noise synthesizer were used. Another problem is the thermal energy resulting from the pressure variation, which heats the crystal, causing its frequency to change. The measurement system then needs to track the crystal frequency by changing the synthesizer frequency. This tracking is performed in discrete steps every 2 seconds or so, and the modulation measurement system must then recover before the next change is made. Heating the crystal to close to its turnover temperature solves this problem.

A wide variety of values of pressure sensitivity have been measured on different crystal types, varying from $1.10^{-9}$ per bar for HC40 crystals with the quad mount [1], to $15.10^{-9}$ per bar for typical TO8 crystals. Some configurations yield pressure sensitivities of as high as $10^{-7}$ per bar.

Typical results for g-sensitivity are shown in Table I, which illustrates the correspondence between the measurement methods. The devices used were some of the units that were reported in [1], and the measurements are of g-sensitivity. The variance in the measurements illustrates the difficulty in achieving repeatable results for devices with sensitivities in this range.

## VII. CONCLUSION

A novel method for characterizing the acceleration sensitivity of precision resonators has been described and compared to other methods. The new method utilizes a vibration table combined with a lock-in amplifier, a variable delay line, and a phase detector to obtain a highly sensitive measurement approach. The advantages over the previously used network analyzer method include an increased measurement resolution, a faster measurement time, and the ability to determine sign. This method was extended to pressure sensitivity by substituting the vibration table and the vibration controller stimulus signal with a sinusoidally modulated pressure chamber and the reference signal derived from a pressure transducer. It is possible to adapt this measurement method to any periodic stimulus to which the frequency of a resonator exhibits sensitivity. Using this method, the measurement resolution for acceleration sensitivity is in the order of a few parts in $10^{12}$ per g and for pressure sensitivity a resolution of approximately one part in $10^9$ per bar can be achieved.

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